ECON42720 Causal Inference and Policy Evaluation

2 Regression Recap and Selection on Observables

Ben Elsner (UCD)

About this Lecture

Linear regression is by far the most important **estimation technique in causal inference**

Yes, I know, machine learning is all the rage these days

- But a linear approximation is often the best we can do
- It's already hard enough to get the linear approximation right
- Fancy techniques are not always better

About this Lecture

Part I: how does regression work? A recap

- Why we use linear regressions and how we estimate them
- The sampling distribution of the OLS estimator

Part II: selection on observables

- Multivariate regression can be used for causal inference
- But for that we need to make strong assumptions

Resources

The material behind these slides can be found in any good econometrics textbook. For introductory econometrics, I recommend

- Wooldridge, J. Introductory Econometrics: A Modern Approach. 7th Edition. South-Western College Publishing, 2019.
- Stock, J. and M. Watson. Introduction to Econometrics. 3rd Edition. Pearson, 2017.

Regression recap

Why do we use linear regression?

- What we want to approximate: the conditional expectation function (CEF)

How does regression work?

- Interpretation of coefficients with and without controls
- Estimation with OLS and Inference

Undergrad Recap: Goal of Linear Regression

Quantify the **expected effect** of a **one unit change in X on Y**

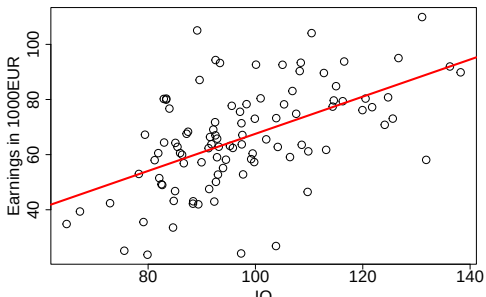
- If X goes up by one unit, by how many units does Y go up or down?
- **Causal interpretation:** If we/nature/an experimenter changes X by one unit, what is the expected effect on Y?

This **effect** is equivalent to the **slope** coefficient β_1 in a **linear regression model**

$$Y = \beta_0 + \beta_1 X + u$$

Linear Regression

Regression analysis means that we **fit a straight line** through (X, Y) data points



In this example, the **regression line** is Earnings $= -3000 + 700$ IQ

- An increase in the IQ by one point increases earnings on average by 700 EUR

What we are looking for: the conditional expectation function

In undergraduate econometrics, you probably learned about a **population regression model**

- the idea is that there is a **true relationship** between X and Y that we want to estimate
- we have a **sample** of n observations from this population and estimate $\widehat{\beta}_0, \widehat{\beta}_1$ using OLS

But is the population regression model (PRM) really what we are looking for?

- Yes and no. The PRM is an approximation of our object of interest
- This object is called the **conditional expectation function** (CEF)

Ingredients: Random Variables

Random variables are variables that take on different values with a certain probability

- x is a random variable that takes on values $x_1, x_2, \ldots, x_n$ with probabilities $f(x_1), f(x_2), \ldots, f(x_n)$

Expected value: the average value of a random variable

$$\begin{aligned} E(x) &= x_1 f(x_1) + x_2 f(x_2) + \cdots + x_k f(x_n) \\ &= \sum_{j=1}^{n} x_j f(x_j) \end{aligned}$$

Expected Value: Example

$x \in \{-1, 0, 2\}$ with probabilities $f(-1) = 0.3, f(0) = 0.3, f(2) = 0.4$. The expected value of X is

$$\begin{aligned} E(x) &= (-1)(0.3) + (0)(0.3) + (2)(0.4) \\ &= 0.5 \end{aligned}$$

Notation

We denote **random variables with lower case letters** x, y.

Typically, we **do not use indices when we talk about the population**. For example, the linear relationship between x and y in the population is

$$y = \beta_0 + \beta_1 x + u$$

Realisations of a random variable are denoted with **lower case letters with indices** x_i with $i = 1, \ldots, n$. We also use indices when

- Talking about **relationships in the sample**
- Talking about particular realisations of a random variable: $x_i = x$, for example $x_i = \textit{female}$ or $x_i = 5$

This can be confusing at the start but you'll get used to it!

The Conditional Expectation Function

We are interested in **explaining the relationship between x and y** in the population

A useful concept in this regard is the **Conditional Expecation Function (CEF)**: $E(y_i|x_i)$

- what is the **population average of y_i for a given value of x_i**
- i.e. what if x_i takes value x?

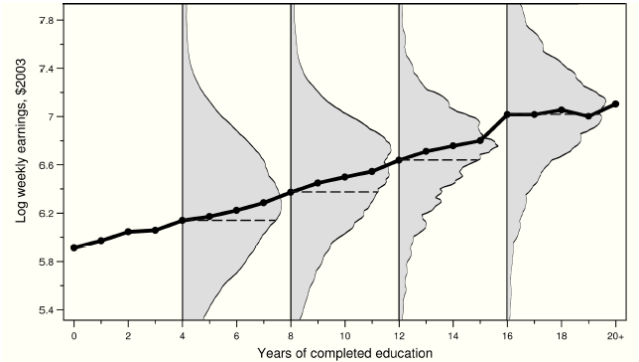
Example: x is a dummy that equals one if a person is female and zero otherwise. y is earnings.

The **CEF can take on two values**:

- Average earnings of women $E(y_i|x_i = 1)$
- Average earnings of men/other $E(y_i|x_i = 0)$

A Continuous CEF

Education vs earnings (from Angrist & Pischke, MHE)



At every level of completed education, we have a different expected value of earnings

- At each value x_i we have a distribution of y_i
- and the CEF comprises the averages of this distribution

Regression is a Linear Approximation of the CEF

The **population regression model** (PRM) is a linear approximation of the CEF

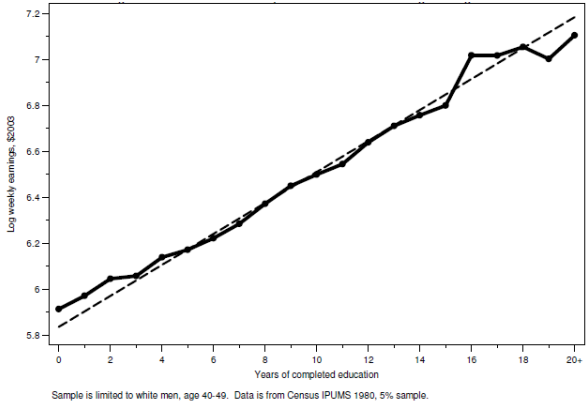
$$y = \beta_0 + \beta_1 x + u$$

- Our **ultimate goal** is to know the **CEF**
- But: with a linear regression, we can estimate the parameters of the PRM
- I.e. we **cannot estimate the CEF directly**

Why a **linear approximation is useful**:

- We typically have small sample sizes, so approximating a non-linear function is difficult
- We are often interested in marginal effects, so a linear approximation is often sufficient

PRM: Linear Approximation of the CEF



The dashed line is the Population regression model $y = \beta_0 + \beta_1 x + u$. The solid line is the CEF $E(y|x)$.

It can be shown that the PRM is the **best linear approximation of the CEF** (see Angrist & Pischke, MHE, ch. 3).

CEF and PRM: what's all this about?‘

The **CEF is the object of interest** in (most of) econometrics

- The PRM $y = \beta_0 + \beta_1 x + u$ is a **linear approximation** of the CEF.
- But it is an approximation, so it can be wrong.
- With data, we can draw inference on the PRM but not on the CEF

What does this mean for the empirical analysis?

- We need to think about the **relationship between** x **and** y in the population
- Linear approximations are more innocuous when we consider small changes in x

The Sample Regression Function

Now suppose we have a **sample of size** n that was **randomly sampled from the population**, $(y_1, x_1), \ldots, (y_n, x_n)$

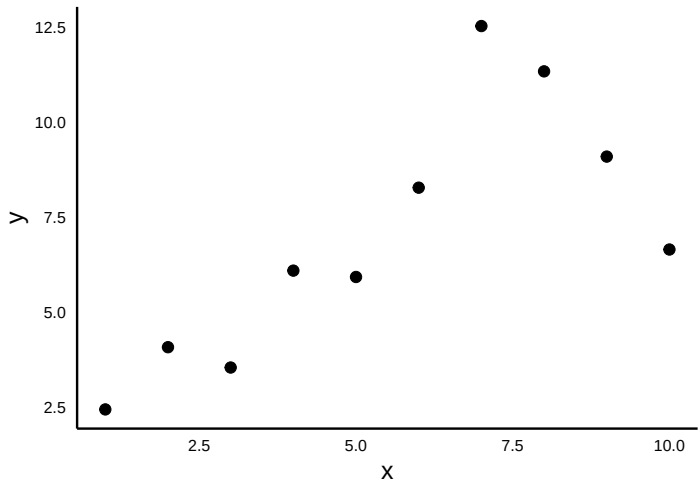
The **sample regression function** is

$$\widehat{y_i} = \widehat{\beta_0} + \widehat{\beta_1} x_i \tag{1}$$

We can estimate the parameters $\widehat{\beta_0}$ and $\widehat{\beta_1}$ with Ordinary Least Squares (OLS)

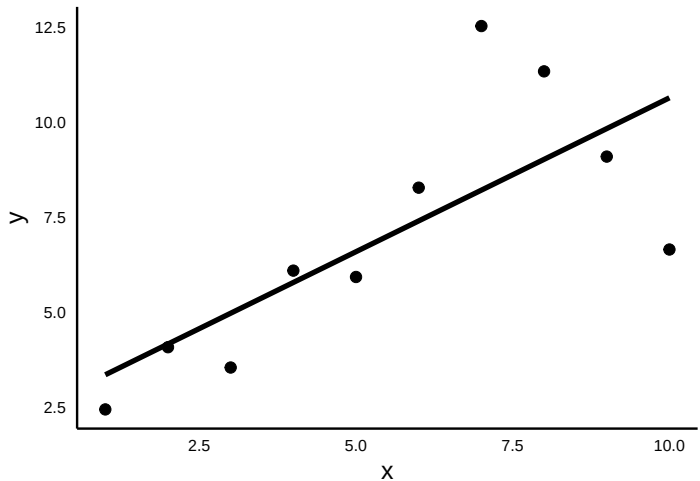
OLS: Intuition

Let's start with some data points



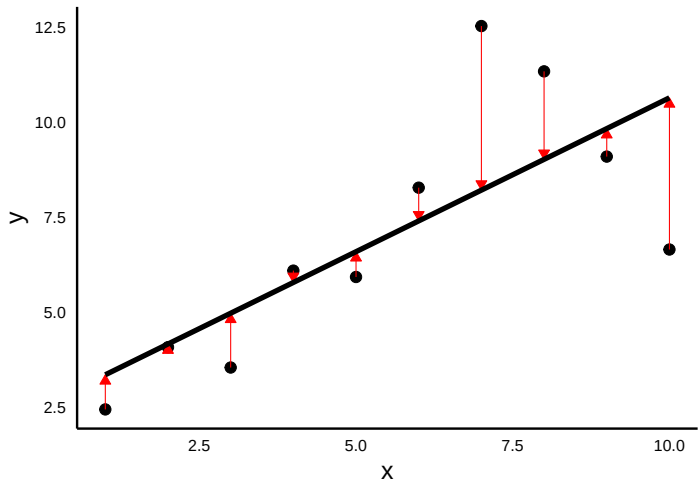
OLS: Intuition

Goal: fit a regression line through those points



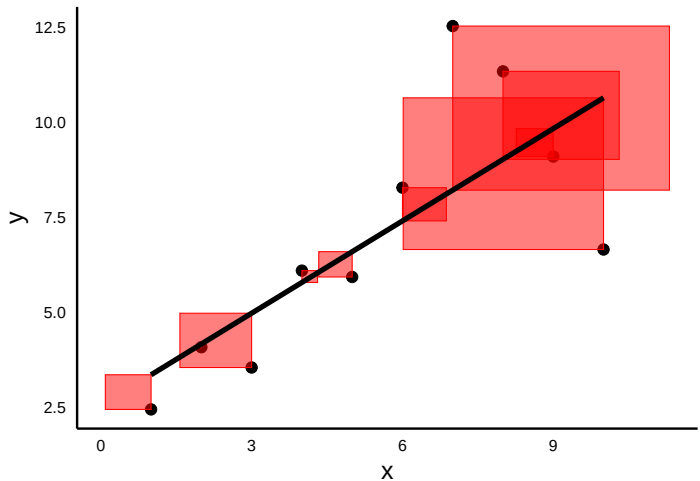
OLS: Intuition

The key ingredient of OLS are the residuals $\widehat{u}_i = y_i - \widehat{b_0} - \widehat{b_1} x_i$



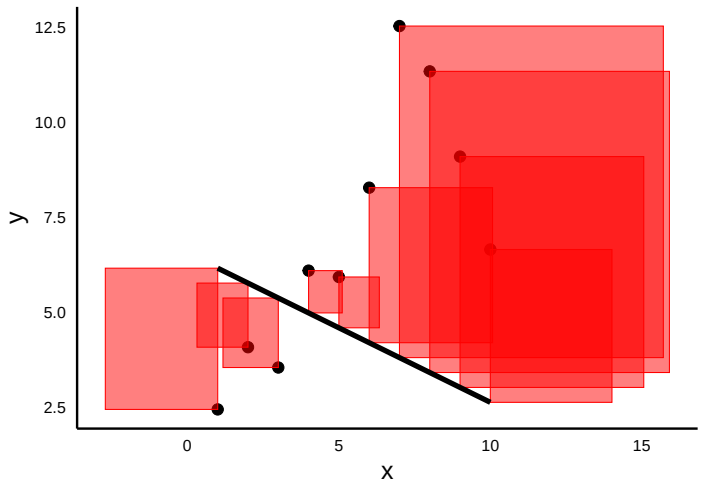
OLS: Intuition

Now consider the square of each residual



OLS: Intuition

Let's consider a different regression line: the squares are much larger!



OLS: Intuition

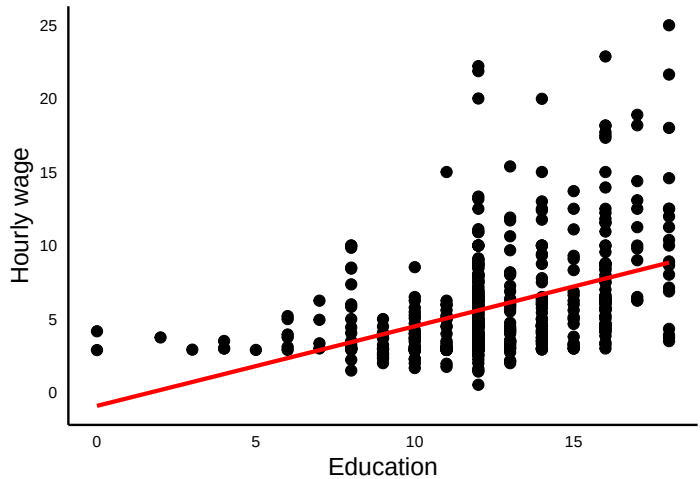
OLS **minimizes the average size of these squares**

It minimizes the sum of squared residuals (SSR)

The result is the **best-fitting line** that describes the relationship between x and y in the sample

OLS: Data Example

Let's look at the relationship between education and wages with data from the U.S.



Regression Output: Interpretation

Table 1: Effect of Education on Wages

	Dependent variable:
	wage
educ	0.541^{***}
	(0.053)
Constant	-0.905
	(0.685)
Observations	526
R^2	0.165
Adjusted R^2	0.163
Residual Std. Error	3.378 (df = 524)
F Statistic	103.363^{***} (df = 1; 524)
Note:	*p<0.1; **p<0.05; ***p<0.01

A 1-year increase in education is associated with a 0.54 USD increase in hourly wages

OLS: The Math

The Ordinary Least Squares (OLS) estimators $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are derived through the minimization problem

$$(\widehat{\beta_0}, \widehat{\beta_1}) = \underset{\widehat{b_0}, \widehat{b_1}}{\arg\min} \sum_{i=1}^{n} [(y_i - \widehat{b_0} - \widehat{b_1} x_i)^2] \tag{2}$$

The sample means $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$ are the sample analogs of the population means $E(y_i)$ and $E(x_i)$

OLS: The Math

The **residuals of the regression** are defined as $\widehat{u_i} = y_i - \widehat{\beta_0} + \widehat{\beta_1} x_i$.

When we "run an OLS regression'', we **minimize the sum of squared residuals (SSR)**, $\sum_{i=1}^{n} \widehat{u_i}^2$ and obtain values for $\widehat{\beta_0}$ and $\widehat{\beta_1}$

Solving the minimization problem (2) yields the **estimators**

$$\begin{aligned} \widehat{\beta_1} &= \frac{\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x})}{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{\widehat{Cov(y_i, x_i)}}{\widehat{V(x_i)}} \\ \widehat{\beta_0} &= \bar{y} - \widehat{\beta_1}\bar{x} \end{aligned} \tag{3}$$

The Sampling Distribution of the OLS Estimator

To **draw inference about the population**, we need to know the **sampling distribution of the OLS estimator**

What we want to know:

- When will $\widehat{\beta}_1$ be **unbiased?**
- What is its **variance?**

To answer these questions, we need to make some **assumptions about the sample and population**

1. Population model is linear in parameters
2. Sample is randomly drawn from the population
3. Variation in x
4. **Zero conditional mean assumption** (ZCM)

OLS Assumptions

The four **OLS assumptions must be fulfilled** for the OLS estimator to be **unbiased and consistent**

Unbiasedness: $E(\widehat{\beta}_1) = \beta_1$

- across many random samples, the estimator gets it right on average

Consistency: $\widehat{\beta}_1 \xrightarrow{p} \beta_1$ as $n \to \infty$

- if the sample size increases, the estimator converges to the true value
- this is a consequence of the Law of Large Numbers (LLN)
- As n gets larger, the sample becomes more representative of the population

The Zero Conditional Mean (ZCM) Assumption

The **ZCM assumption is the most important assumption** in this module

- It is **not testable with the data** at hand
- It rarely holds in practice (except in randomised experiments)
- Causal inference techniques exploit scenarios where ZCM holds approximately

Other names for the ZCM assumption:

- **Conditional independence assumption (CIA)**
- **Exogeneity assumption**

The Zero Conditional Mean (ZCM) Assumption

Consider the **population model**

$$y = \beta_0 + \beta_1 x + u$$

The **ZCM assumption** states that the conditional mean of the error term is zero

$$E(u|x) = E(u) = 0$$

What does this mean?

- The error term is **not systematically related** (speak: uncorrelated) with x
- At any level of x, the average value of u is zero

ZCM Assumption: Example

Does **higher education (causally) increase earnings?**

$$wage_i = \beta_0 + \beta_1 education_i + u_i$$

What is the error term u_i here?

- Any determinant of a person's wage that is not education
- E.g., innate ability, motivation, personality, etc.

ZCM Assumption: Example

Say u_i includes ability. According to the ZCM assumption, the following must hold:

$$E(\text{ability} \mid \text{education} = 8) = E(\text{ability} \mid \text{education} = 12) = E(\text{ability} \mid \text{education} = 16)$$

So it **must hold that**:

- the average ability of people with 8 years of education is the same as
- the average ability of people with 12 years of education and
- the average ability of people with 16 years of education

This is hardly plausible $\Rightarrow$ **ZCM assumption is violated**

What if ZCM is Violated?

The OLS estimator of β_1 is **biased and inconsistent**

Bias: $E(\widehat{\beta_1}) \neq \beta_1$

- The expected value of the OLS estimator is not equal to the true value of β_1
- Across many samples, the estimates are systematically too big or too small

Inconsistency: $\widehat{\beta_1}$ does not converge to β_1 as $n \to \infty$

- Even if the sample size is very large, the OLS estimator does not converge to the true value of β_1

How to think about Violations of ZCM

The variable x is **typically a choice**

- People choose how much education to get, how often they go to the gym, how much they save, who they want to date, etc
- Firms choose how much to invest, how many workers to hire, how much to pollute, etc.
- Governments choose how much to spend on education, how much to tax, etc.

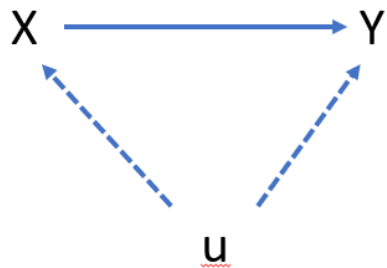
The **choice** of x is typically **influenced by other factors** u

- Individual factors: ability, motivation, preferences, etc.
- Firm factors: technology, market conditions, etc.
- Government factors: ideology, political pressure, etc.

Problem: u **affects** y not just through x but also **through other paths** or directly

How to think about Violations of ZCM

The error term u includes **one or more confounders**



Here u includes a confounder y directly and through x

Example: x is education, y is earnings, u is ability

Omitted Variable Bias

Suppose the **true model** is $y = \beta_0 + \beta_1 x + \beta_2 s_1 + e$

However, we **estimate the model** $y = \tilde{\beta}_0 + \tilde{\beta}_1 x + u$

It can be shown that the **OLS estimator is biased**

$$\tilde{\beta}_1 = \beta_1 + \underbrace{\beta_2 \frac{Cov(x, s_1)}{Var(x)}}_{OVB}$$

So when does ZCM hold?

ZCM holds if x is **as good as randomly assigned** to individuals

- This is the case if x is assigned in a **randomised experiment**
- Or if x is assigned in a **quasi-experiment** that mimics random assignment
- Or if we can **control for all confounders** in the analysis

We should **always assume that ZCM is violated**. Researchers need to **think hard about confounders and how to eliminate them.**

The Sampling Distribution of the OLS Estimator

So far, we only talked about the **mean of the OLS estimator** $\widehat{\beta}_1$

- Suppose we run the **same regression in many samples**
- The **mean of the OLS estimator** is the **average** of the estimates **across samples**

However, to draw **inference about the population**, we need to know more than the mean

- How is $\widehat{\beta}_1$ **distributed across samples?**
- But how can we know this distribution if we only have one sample?

The Sampling Distribution of the OLS Estimator

Enter: the **Central Limit Theorem (CLT)**

- If the **sample size is large enough**...
- the **sampling distribution is approximately normal**

Under the CLT and an assumption of homoskedastic errors, the sampling distribution of the estimator is:

$$\widehat{\beta}_1 \sim N(\beta_1, \sigma^2_{\widehat{\beta}_1})$$

where $\sigma^2_{\widehat{\beta}_1} = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$

Going into the assumptions behind this distribution is beyond this module. Most econometrics textbooks provide in-depth discussions

The Sampling Distribution of the OLS Estimator

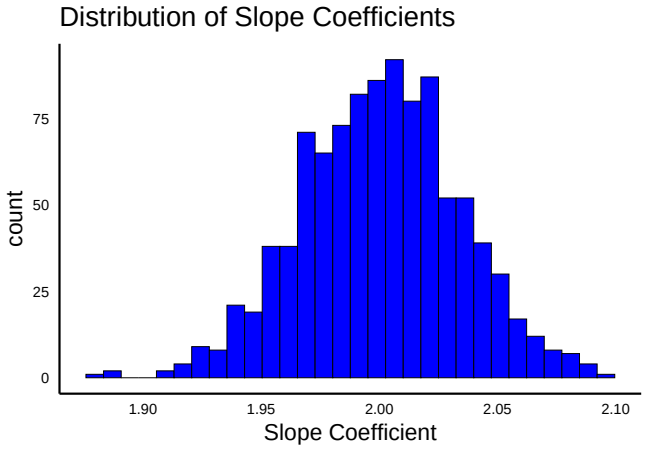
Let's draw 1,000 samples of size $n = 100$ from the following data-generating process and run 1,000 regressions:

$$y = 1 + 2x + u$$

where $u \sim N(0, 1)$ and $x \sim U(0, 10)$

And then we run a regression in each sample and store the slope coefficient $\widehat{\beta}_1$

The Sampling Distribution of the OLS Estimator



- The distribution is centered on the true value $\beta_1 = 2$
- The distribution is approximately normal

The Sampling Distribution of the OLS Estimator

An important statistic for inference is the **standard error** of the OLS estimator

$$SE(\widehat{\beta}_1) = \sqrt{\frac{\widehat{\sigma}^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

where $\widehat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n}\widehat{u}_i^2$

$SE(\widehat{\beta}_1)$ is an estimate of the **standard deviation of the sampling distribution**

- It tells us **how much the he OLS estimator varies across samples**
- **Larger sample size** $\rightarrow$ smaller standard error

Why We Need the Sampling Distribution

We **need the sampling distribution** to draw **inference about the population**

- We typically **test the hypothesis that the true value** $\beta_1 = 0$
- Based on the (estimated) sampling distribution, we can test this hypothesis
- We can use the **t-statistic**, the **p-value**, or **confidence intervals**

How this works: please check econometrics/statistics textbooks

Controlling for Confounders: Multivariate Regression

We can include **confounders in the regression model** to control for them

$$y = \beta_0 + \beta_1 x + \boldsymbol{S}\boldsymbol{\gamma} + u$$

Here, $\boldsymbol{S}$ is a vector of covariates $\boldsymbol{S} = (s_1', s_2', \ldots, s_k')$ and $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_k)$ is a vector of coefficients,[1] i.e.

$$\boldsymbol{S}\boldsymbol{\gamma} = \gamma_1 s_1 + \gamma_2 s_2 + \cdots + \gamma_k s_k$$

We are **only interested in β_1, the causal effect of x on y**

- The other coefficients $\gamma_1, \gamma_2, \ldots, \gamma_k$ are not of interest (nuisance parameters)
- We include the covariates $\boldsymbol{S}$ to control for confounders

[1] Note: each element of $\boldsymbol{S}$ is in itself an $(1 \times n)$ vector. E.g. $s_1 = (s_{11}, s_{12}, \ldots, s_{1n})$ so $\boldsymbol{S}$ is actually an $(n \times k)$ matrix

Interpretation of β_1 in Multivariate Regression

β_1 now has a **ceteris paribus interpretation**

- **Holding all other variables *S* constant**, a one unit increase in x leads to a β_1 unit increase in y

The **inclusion of *S*** allows for a **like-with-like comparison**

- We **compare units with the same values** of ***S*** but different values of x
- But the like-with like comparison is only valid if ***S*** contains all confounders

Anatomy of Regression: the Frisch-Waugh-Lovell Theorem

Population model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

The FWL theorem states that three OLS estimators for β_1 are equivalent

1. regressing y on x_1 and x_2
2. regressing y on $\tilde{x}_1$, the residuals from a regression of x_1 on x_2
3. regressing $\tilde{y}$ on $\tilde{x}_1$, with $\tilde{y}$ being the residuals from a regression of y on x_2

The main insight comes from point 2...

Anatomy of Regression: the Frisch-Waugh-Lovell Theorem

Multivariate regression does two steps at the same time:

1. It purges the correlation between x_1 and x_2 from x_1; the residuals $\tilde{x}_1$ are "what's left"; they are uncorrelated with x_2
2. It estimates the effect of $\tilde{x}_1$ on y, i.e. *after* purging the correlation between x_1 and x_2

Why is this important?

- If x_2 is a **confounder**, we can **purge** its influence by including it in the regression

The same also holds for more than one control variable

FWL Example: Effect of Education on Earnings

Suppose `abil` (ability) is a confounder

Let's first look at the auxiliary regressions (for generating the residuals)

Table 2:

	Dependent variable:	
	educ	wage
	(1)	(2)
abil	1.279***	1.123***
	(0.039)	(0.032)
Constant	4.459***	2.446***
	(0.258)	(0.212)
Observations	526	526
R^2	0.670	0.699
Adjusted R^2	0.669	0.699
Note:	*p<0.1; **p<0.05; ***p<0.01	

FWL Example: Effect of Education on Earnings

Now let's look at the main regressions

Table 3:

	Dependent variable:			
		wage		residwage
	(1)	(2)	(3)	(4)
educ	0.764***	0.533***		
	(0.017)	(0.027)		
abil		0.442***		
		(0.043)		
resideduc			0.533***	0.533***
			(0.061)	(0.027)
Constant	−0.036	0.072	9.562***	0.000
	(0.221)	(0.202)	(0.097)	(0.043)
Observations	526	526	526	526
R^2	0.790	0.826	0.127	0.421
Adjusted R^2	0.790	0.825	0.125	0.420
Note:			*p<0.1; **p<0.05; ***p<0.01	

Insights from the previous slide

Columns (1) and (2) show that the **omission of `abil` leads to an upward bias**

The **coefficient of `educ` is the same** in Columns (2), (3) and (4) is the same $\Rightarrow$ **FWL**

The regression in Column (2) takes **two steps at a time**:

1. **"regresses out"** (a.k.a "partials out") the **confounding influence** of ability on education
2. provides the **partial effect of education on wages** that is not driven by ability

Thanks to FWL, we can **eliminate the confounding influence of other variables**. This is **BIG**!

Conditional Mean Independence Assumption

The **Conditional Mean Independence Assumption (CMIA)** is a "light" version of the ZCM assumption.

$$E(u \mid x, \boldsymbol{S}) = E(u \mid S) = 0$$

In plain English: as long as $\boldsymbol{S}$ **is included, the error term** u **is uncorrelated with** x

- x is exogenous conditional on $\boldsymbol{S}$
- x is as good as random conditional on $\boldsymbol{S}$

Controlled Regression in Practice

Elsner & Wozny (2023): what is the **impact of low-dose radiation on cognitive ability?**

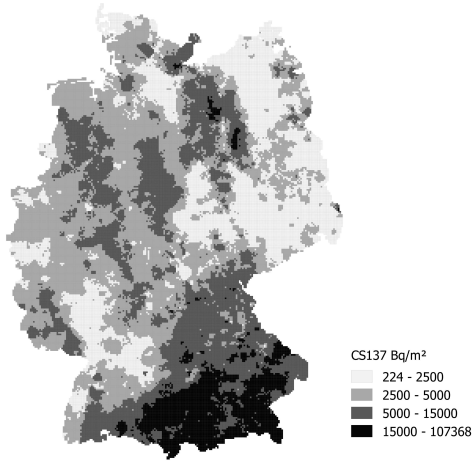
- Long-run exposure to low-dose radiation can affect cognitive ability through its effect on the brain and general health effects

Challenge: people are usually not randomly exposed to radiation

- Our setting: exploit variation in the Chernobyl fallout across Germany
- This was driven by wind and rain patterns in April 1986
- Look at the effect on test scores 25 years

Variation in Chernobyl-induced ground radiation in Germany, 1986

Is this variation as good as random?



Example: Elsner & Wozny (2023)

$$testscore_i = \beta_0 + \beta_1 radiation_i + u_i$$

Radiation exposure is likely not random...

- People could not foresee the Chernobyl accident
- But: some areas are more prone to radiation when an accident happens
- More prone: higher rainfall, higher altitude, further East
- People in more prone areas are likely different from people in less prone areas

So **ZCM is violated**: $E(u \mid radiation) \neq 0$

Solution: include potential confounders

We need to **control somehow for "proneness to radiation"**

- Control for average rainfall in April and for altitude

We can **include state dummies**

- Compare **people who reside in the same state** but are exposed to **different levels of radiation**
- This means that we run a **regression with group fixed effects δ_s** (more on this later)

Solution: include potential confounders

$$testscore_i = \beta_0 + \beta_1 radiation_i + \beta_2 rain_i + \beta_3 alt_i + \boldsymbol{\delta_s} + u_i$$

Does the Conditional Mean Independence Assumption (CMIA) hold?

- I.e. is radiation as good as randomly assigned conditional on rainfall, altitude and state dummies?

$$E(u_i \mid radiation_i,\ rain_i,\ alt_i,\ \boldsymbol{\delta_s}) = 0$$

Does CMIA Hold? Balancing Tests

We run **balancing tests based on observable characteristics** (age, gender, education, parental education)

- **Run regressions** of observable characteristics on radiation, rainfall, altitude and state dummies
- Ideally, there should be **no correlation between radiation and the observables**
- This would be **evidence in favour of CMIA**, but no proof

Results:

- radiation is correlated with some observables: negatively with own and parental education
- but this correlation is **removed once we control for altitude and rainfall**
- this is evidence that CMIA holds

See here for the table of balancing tests: LINK

Results

Table 3
The effect of radiation on cognitive performance — OLS and IV results.

	Math	Reading	List. comp.	ICT	Scient. lit.	Reas.
A. OLS Estimates						
OLS coefficient	−0.011***	−0.013**	−0.008*	−0.005	−0.004	−0.000
	(0.003)	(0.006)	(0.004)	(0.004)	(0.003)	(0.004)

Reas.	Read. speed	Percp. speed	Cog index
−0.000	−0.007*	−0.004	−0.008**
(0.004)	(0.004)	(0.003)	(0.003)

Higher radiation in 1986 $\Rightarrow$ **lower test scores** in 2011

In the paper we do a lot more to convince the reader

Using Controlled Regression for Causal Inference

The **key to any research design** is **CREDIBIILTY**

With controlled regression, the credibility of causal identification hinges on the ZCM or CMIA assumption

- Assumption: conditional on controls, x is as good as randomly assigned
- This design is called **selection on observables**
- The assumption is untestable! But we can (and must) bring evidence and good arguments in favour of it

Selection on observables: ingredients of a good paper

1) Theory: why should x affect y to begin with? And through what channels?

2) A **clear statement and discussion of the identification assumption**:

- What is the ZCM/CMIA in your context? Why should it hold? What are potential challenges?
- DAG: what are the confounders? Why should I include them in the regression?

3) Balancing tests:

- show that x is as good as randomly assigned conditional on controls

4) Extensive **robustness checks** to rule out alternative explanations

Common mistake: kitchen sink regressions

A **common problem**: people start with a model like this

$$y_i = \beta_0 + \beta_1 x_i + \boldsymbol{S}\boldsymbol{\gamma} + u_i$$

and include in $\boldsymbol{S}$ **every variable they can find** (everything but the kitchen sink)

Don't do this!!!

- you could condition on colliders or mediators
- you could condition on irrelevant variables
- readers won't believe you

DAGs will help!

Summary: What you need to understand for this module

Logic of linear regression

- Why we use linear regression
- Why and how we use OLS to estimate the parameters of the linear regression model
- How to interpret the OLS estimator $\hat{\beta}_1$
- What multivariate regression does, especially the FWL theorem

Uses and limitations of linear regression for causal inference

- The ZCM assumption is violated in most applications, leading to OVB
- How control variables can be used to control for confounders: the CMIA assumption

References

Elsner, Benjamin, & Wozny, Florian. 2023. Long-run exposure to low-dose radiation reduces cognitive performance. *Journal of Environmental Economics and Management*, **118**, 102785.

Appendix

Regression with R

```r
# Required packages (install if necessary)
library(tidyverse)
library(wooldridge)
library(stargazer)
```

Regression with R

This code shows how to estimate and present regressions with R

```
data('wage1') # load the data
df <- wage1
reg1 <- lm(wage ~ educ, data = df) # estimate simple regression
reg2 <- lm(wage ~ educ + exper, data = df) # estimate multivariate regress

stargazer(reg1, reg2, type = "text") # print regression results
```

Regression outputs with Stargazer and R Markdown

You can generate nice regression tables with the `stargazer` package and R Markdown/Quarto. Here is a code chunk that gives you a nicely formatted latex table.

```
```{r, results='asis', echo=FALSE}
stargazer(reg1, reg2,
      header=FALSE,
      type='latex',
      title="Effect of Education on Wages")
```
```

Regression with R

We can also generate a scatter with a regression line

```
ggplot(df, aes(x = educ, y = wage)) + # generate scatter plot
  geom_point() + # add points
  geom_smooth(method = "lm", se = FALSE) +  # add regression line
  theme_minimal()
```

Group Work Questions I

What is the difference between unbiasedness and consistency of an estimator?

What is the sampling distribution of an estimator? Provide an intuitive explanation

How is the sampling variance of an estimator related to the sample size?

Group Work Questions II

Suppose you have a variable z that is omitted from the population model

$$y = \beta_0 + \beta_1 x + \beta_2 z + u$$

The correct population parameter, β_1, is positive.

- a) Suppose you know that $cov(x, z) > 0$ and $\beta_2 < 0$. How will the OLS estimator $\widehat{\beta}_1$ be biased when z is omitted? Will it be over- or underestimated?
- b) Draw a DAG with z as a confounder and indicate the two factors that make up the omitted variable bias

Group Work Questions III

On the following slide, you see a regression output with two columns. In data on school districts in California, we want to study whether larger class sizes lead to lower test scores. The dependent variable is the average test score in a school district; the regressor of interest is the average class size in a school district. In Column (2), we control for the percentage of students in a school district who are not English native speakers (a proxy for poverty).

- a) Interpret the slope coefficient on class size in Column (1)
- b) Given the results in Column (2), what can we say about the correlation between class size and the share of non-native English speakers in a school district?

Group Work Questions III cont'd

Table 4:

	Dependent variable:	
	testscr	
	(1)	(2)
str	−2.280***	−1.101***
	(0.480)	(0.380)
el_pct		−0.650***
		(0.039)
Constant	698.933***	686.032***
	(9.467)	(7.411)
Observations	420	420
R^2	0.051	0.426
Adjusted R^2	0.049	0.424
Note:	*p<0.1; **p<0.05; ***p<0.01	

benjamin.elsner@ucd.ie

www.benjaminelsner.com

Sign up for office hours

YouTube Channel

@ben_elsner

LinkedIn

Contact

Prof. Benjamin Elsner
University College Dublin
School of Economics
Newman Building, Office G206
benjamin.elsner@ucd.ie

Office hours: book on Calendly